An Orbital Determination of Triton with the Use of a Revised Pole model

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1 INTRODUCTION

The giant planets Jupiter, Saturn, Uranus and Neptune with their respective satellites form some micro-solar systems in which various gravitational, orbital and physical problems of interest are similar. These small solar systems constitute several natural laboratories for the study of the formation and evolution of the Solar system. Meantime, the researches of natural satellites motion can greatly facilitate the improvement of ephemeris for major planets. We initiated an astrometric observing programme of natural satellites in 1985. Some results of our observations (Qiao, et al. 2007, 2008, 2011, 2013) have been already used to develop new orbits for satellites, such as the eight main satellites of Saturn (Harper et al., 1988; Dourneau, 1993; Harper & Taylor, 1993), Phoebe the ninth satellite of Saturn (Shen et al. 200,5; Emelyanov, 2007; Desmars, et al., 2013), the major satellites of Uranus (Emelyanov & Nikonchuk, 2013) and Triton the main satellite of Neptune (Jacobson 2009; Zhang et al. 2014).

Triton, the largest satellite of Neptune, was discovered by the British astronomer William Lassell using telescope on October 10, 1846. The Neptunian tidal friction can affect the motion of Triton by transferring angular momentum between the orbiting Triton and the spinning Neptune. Earlier orbits for Triton were given by Eichelberger & Newton (1926) and by Harris (1984). In those models, an inclined orbit precessing at a constant rate was adopted to represent Triton's motion. To date, the best available orbit of Triton was completed by Jacobson (1990a, 2009) and by Jacobson, Reidel & Taylor (1991) in employing a precessing pole model of Neptune (Jacobson 1990b). These previous works used the observations over a century, containing the Earth-based visual, photographic and CCD observations with also some spatial observations from radio tracking of the Voyager spacecraft. In the Neptunian system, the oblateness force depends upon the orientation of the pole of Neptune. The polar motion is driven primarily by the torque due to the gravitational attraction of the Triton on the planet's equatorial bulge, which causes the orbit to actually precess at constant inclination to a plane about the Neptune pole. The significant feature of the perturbational model of Triton arises more complicated calculations than for other satellites.

2 DYNAMICAL MODEL

In the force model, we have included the following forces: the central force of the primary; the perturbing force due to the Sun, Saturn, Jupiter and Uranus; the perturbation due to the Neptunian oblateness which is related to the orientation of the pole of Neptune that precesses and rotates at constant rate about angular momentum vector of the Neptunian system.

In this work, we use a revised pole model presented here for a better representation of the pole direction with time than previous Peters (1981) representation which could not be valid over the whole time span of the observational data (Jacobson 1990a). Moreover, we checked such a better validity of the revised pole adopted here as we obtained a better convergence in Triton's orbit than in using Peters (1981) formulae. Here, we derive the accelerations and partial derivatives of the acceleration upon a satellite due to the oblateness of Neptune in an arbitrary planetocentric reference.

We suppose that the direction cosines of the pole vector of the planet are defined as $(\gamma_1, \gamma_2, \gamma_3)$, and that the planetocentric coordinates of the satellite are (x_1, x_2, x_3), then

$$r^2 = x_1^2 + x_2^2 + x_3^2 \qquad \zeta = \frac{\gamma_1 x_1 + \gamma_2 x_2 + \gamma_3 x_3}{r}$$

The potential function for the effect of the nth zonal harmonic of the gravity field the planet upon the satellite is

$$\phi = \frac{k(\mu_0 + \mu)}{r} \sum_{n=2}^{4} J_n \left(\frac{a_0}{r}\right)^n P_n(\zeta) = K \frac{1}{r^{n+1}} \sum_{n=2}^{4} J_n P_n(\zeta)$$

where $K = -k(\mu_0 + \mu) J_n a_0^n$, μ_0 is the mass of the planet, a_0 is the mass of satellite, is the equatorial radius of the planet and J_n is the coefficient of the nth zonal harmonic.

The acceleration component of coordinate x_i is

$$F_i = \frac{\partial \phi}{\partial x} = \frac{K}{r^n + 2}\left(-\frac{x_i}{r} P'_{n+1}(\zeta) + \gamma_i P'_n(\zeta)\right)$$

where we have used the identity

$$(n+1) P_n(\zeta) + x P'_n \equiv P'_{n+1}(x)$$

The partial derivative of F_i with respect to x_j is obtained after algebraic process and the use of another identity in Legendre polynomial. It is found to be

$$\frac{\partial F_i}{\partial x_i} = \frac{K}{r^{n+3}}\left((-\delta_{ij} + (n+3)\xi_i \xi_j) P'_{n+1}(\zeta) + \gamma_i \gamma_j P''_n(\zeta) + (\xi_i \xi_j \zeta - \xi_i \gamma_j - \xi_j \gamma_i) P''_{n+1}(\zeta)\right)$$

where $\xi_k = x_k / r$.

3 COMPARISON BETWEEN TWO EPHEMERIDE WITH THE NSCD DATA

The observations applied in our calculation are taken fromNSDC, as shown in Table 1, in which columns listed contain: period, observer, type of observation and number of observation. In the last two columns also are listed the rms of the post-fit observation residuals and their means. These observations include all of the available observations after 1970 modern observing development, in which a large amount of observation is astrometric accurate CCD observations.

The new orbit will be available for the scientific community on the Saimirror MULTI-SAT server of the IMCCE (Emel'Yanov & Arlot 2008) at the following address: www.imcce.fr/hosted_sites/saimirror/nssephf.php.

Table1. List of all the series of observations used here to fit our new orbit of Triton

Obs. code	Period	Observer	Type	N	O – C(Z) σ (″)	O – C(Z) μ (″)	O – C(J) σ (″)	O – C(J) μ (″)
689	1975-1977	USNO	Phot.X	28	0.0707	0.0286	0.0714	0.0270
			Phot.Y	28	0.0874	-0.0419	0.0871	-0.0398
689	1979-1983	USNO	ccd.X	114	0.0385	-0.0019	0.0359	0.0002
			ccd.Y	114	0.0721	-0.0246	0.0711	-0.0242
689	1984-1986	Flagstaff	Phot.X	56	0.0280	0.0037	0.0233	0.0062
			Phot.Y	56	0.0300	-0.0651	0.0257	-0.0659
119	1986-1993	Abastumani	Phot.α	54	0.4521	-0.0012	0.4521	0.0017
			Phot.δ	54	0.4098	-0.0748	0.4133	-0.0753
874	1989-1994	Veiga	ccd.X	433	0.4563	0.0914	0.4561	0.0904
			ccd.Y	433	0.2009	0.0355	0.1972	0.0342
188	1990	Majdanak	Phot.α	5	0.0912	0.1061	0.0958	0.1081
			Phot.δ	5	0.0410	0.3043	0.0396	0.3134
874	1995-1997	Veiga	ccd.X	759	0.1265	-0.0937	0.1247	-0.0941
			ccd.Y	759	0.1948	-0.0882	0.1928	-0.0878
337	1996-2006	Qiao	ccd.α	943	0.0570	0.0813	0.0574	0.0829
			ccd.δ	943	0.0411	-0.0256	0.0437	-0.0282
689	1998-2000	Flagstaff	ccd.α	188	0.1270	-0.0139	0.1270	-0.0137
			ccd.δ	188	0.1142	-0.0172	0.1136	-0.0198
673	1999,2001	Table Moun	ccd.△α	6	0.1003	0.0792	0.1010	0.0782
			ccd△δ	6	0.0351	-0.0037	0.0395	0.0019
673	1999,2001	Table Moun	ccd.α	6	0.1656	-0.0920	0.1705	-0.0930
			ccd. δ	6	0.0972	-0.0960	0.1045	-0.0904
874	2000-2002	Itajuba	ccd.α	66	0.1590	-0.0608	0.1588	-0.0571
			ccd. δ	66	0.2440	0.0052	0.2447	0.0060
689	2001-2005	Flagstaff	ccd.α	323	0.1151	-0.0056	0.1156	-0.0052
			ccd. δ	323	0.1213	-0.0303	0.1224	-0.0307
689	2005-2006	Flagstaff	ccd.α	144	0.1372	-0.0096	0.1376	-0.0088
			ccd. δ	144	0.1455	0.2056	0.1473	0.2039

4 COMPARISON BETWEEN TWO EPHEMERIDES WITH THE NEW OBASERVATIONS (2007-2009)

As a continuation of our previous observing campaign of 1996–2006 (Qiao, et al. 2007), we present here another1095 new observed positions of Triton which were obtained by using three different telescopes at two different stations during the period 2007–2009, spreading over 46 nights involving eight missions. For more instrumental details concerning the CCD detectors and the reflectors, see Table 2.

Table 3 shows the results of the comparison. These residuals appear to be quite similar to those derived from Jacobson (2009), within 1 mas for the mean residuals, corresponding to only 20 km in the position of Triton. This shows that both of the two orbits by Jacobson (2009) and by Zhang et al. (2014) can be considered as equivalent for the recent period of our observations from 2007 to 2009.

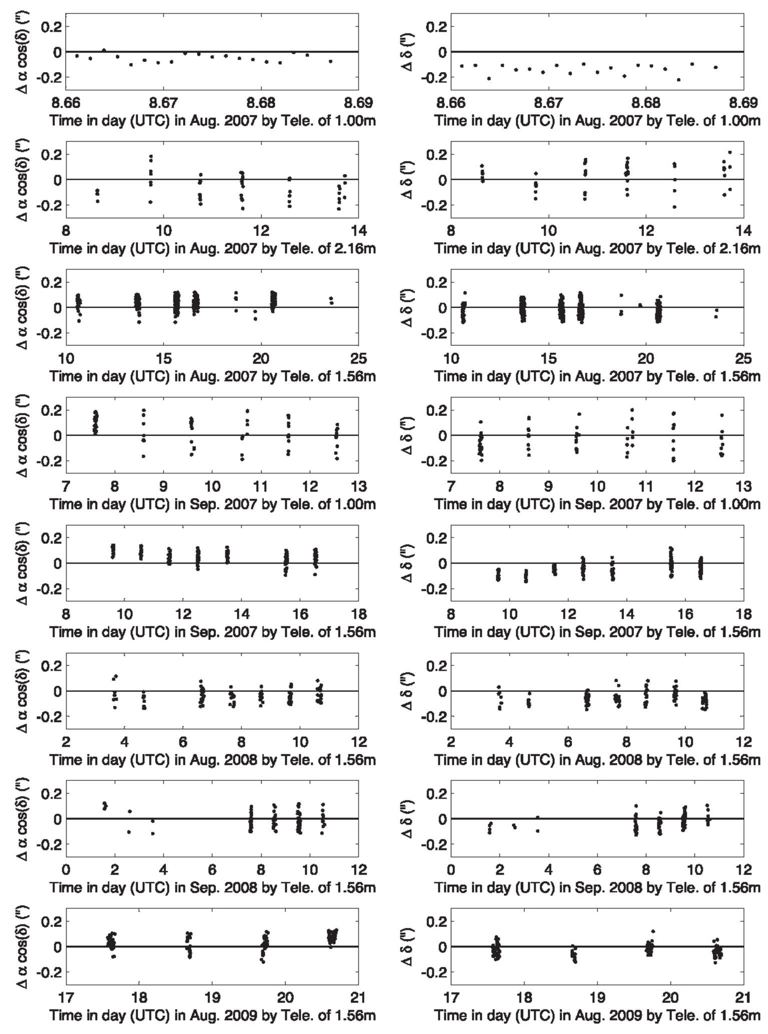
Table 2. Specifications of the three telescopes and CCD chips used for the observations of Triton.

Telescope	A	B	C
Diameter of primary mirror	156 cm	100 cm	216 cm
Focal length	15 600 mm	7800 mm	10 000 mm
Size of CCD array (pixels)	2112 × 2048	1340 × 1300	2080 × 2048
Size of pixel	24 μm	20 μm	15 μm
Angular extent per pixel	0.32 arcsec	0.53 arcsec	0.31 arcsec
Field of view	11.2 arcmin × 10.8 arcmin	11.8 arcmin × 11.4 arcmin	10.7 arcmin × 10.5 arcmin
Bandpass of CCD (nm)	300–1100	200–1100	300–1000

Table 3. The mean residuals μ(arcsec) and standard deviations σ(arcsec) of theO−C residuals of the comparison between our observations and the theoretical positions.

Telescope	Mission	N.(d)	N.(Images)	Jacobson (2009)+ DE431 μ_α(arcsec)	μ_δ(arcsec)	σ_α(arcsec)	σ_δ(arcsec)	Zhang et al. (2014)+DE431 μ_α(arcsec)	μ_δ(arcsec)	σ_α(arcsec)	σ_δ(arcsec)
A (1.56 m)	2007 Aug.	8	429	0.043	−0.016	0.041	0.052	0.048	−0.022	0.043	0.056
	2007 Sep.	7	183	0.050	−0.044	0.045	0.051	0.048	−0.037	0.047	0.052
	2008 Aug.	7	109	−0.043	−0.057	0.053	0.051	−0.046	−0.055	0.055	0.050
	2008 Sep.	7	102	−0.018	−0.027	0.064	0.052	−0.025	−0.016	0.067	0.056
	2009 Aug.	4	133	0.041	−0.033	0.054	0.042	0.045	−0.038	0.053	0.043
B (1.00 m)	2007 Aug.	1	19	−0.050	−0.139	0.031	0.038	−0.044	−0.149	0.033	0.039
	2007 Sep.	6	70	0.036	−0.040	0.106	0.103	0.038	−0.042	0.112	0.106
C (2.16 m)	2007 Aug.	6	50	−0.068	0.029	0.095	0.100	−0.072	0.031	0.099	0.103
Total (A+B+C)	8	46	1095	0.023	−0.029	0.067	0.062	0.024	−0.030	0.071	0.065

Figure 1. Residuals (O−C) of Triton observations in 2007–2009, derived from the comparison of all our observations to the Triton orbit model (Jacobson (2009) + DE431 .



Also, in Fig 1, we have plotted the residuals versus time for each of the eight missions from 2007 to 2009. Fig. 1 visualizes and confirms the different levels of accuracy of each used instrument that we have evaluated and discussed just above from the values of Table 3.

5 COMPARISON BETWEEN THE DIFFERENT EPHEMERIDES OF NEPTUNE

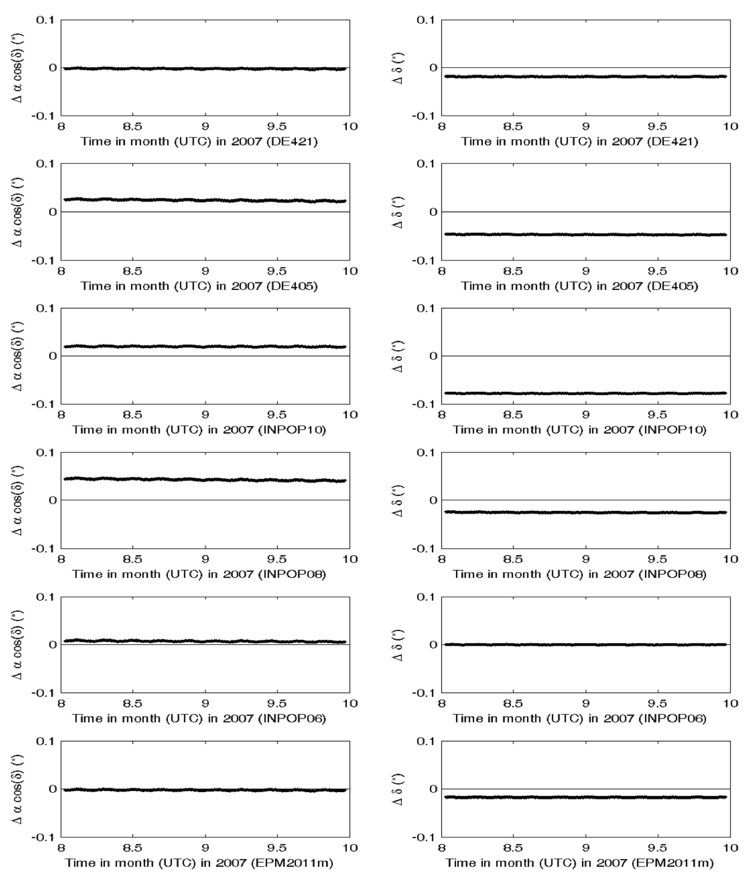
In order to compare different ephemerides and to evaluate their respective reliability. we have considered a total of 10 different ephemerides of Neptune to be compared now. The results are presented in Table 4.

Table 4. Mean residuals μ(arcsec) and standard deviations σ(arcsec) of the O−C residuals derived from the comparison of our observations to the theoretical positions of Triton successively obtained from 10 different planetary ephemerides of Neptune and from the two orbits of Triton.

No.	Planetary ephemerides	Triton model: Jacobson (2009) μ_α(arcsec)	μ_δ(arcsec)	σ_α(arcsec)	σ_δ(arcsec)	Triton model: Zhang et al. (2014) μ_α(arcsec)	μ_δ(arcsec)	σ_α(arcsec)	σ_δ(arcsec)
1	DE431	0.023	−0.029	0.067	0.062	0.024	−0.030	0.071	0.065
2	DE421	0.022	−0.048	0.070	0.062	0.022	−0.049	0.071	0.065
3	DE405	0.051	−0.076	0.069	0.062	0.051	−0.077	0.070	0.065
4	DE406	0.051	−0.076	0.069	0.062	0.051	−0.077	0.070	0.065
5	DE200	−0.900	−0.390	0.073	0.065	−0.900	−0.391	0.074	0.067
6	INPOP10	0.044	−0.109	0.069	0.063	0.045	−0.110	0.070	0.065
7	INPOP08	0.072	−0.053	0.069	0.062	0.072	−0.054	0.070	0.065
8	INPOP06	0.033	−0.028	0.069	0.062	0.033	−0.029	0.070	0.065
9	EPM2011m	0.022	−0.047	0.070	0.062	0.022	−0.048	0.071	0.065
10	VSOP87	−0.816	−0.361	0.073	0.064	−0.815	−0.362	0.074	0.067

In Fig 2, we present the differences between the theoretical positions of Triton obtained from DE431 and from each of the other planetary ephemerides.

Figure 2. Differences between the positions of Triton successively obtained from different planetary ephemerides.



CONCLUSION

We have presented a new determination of the orbit of Triton. The orbit has been checked with some comparisons from Jacobson (2009) with all the available observations spreading over the period 1975–2006, and then with the recent period of our observations. They provide the same values of mean residuals, within 1 mas, corresponding to only 20 km in the position of Triton.

Moreover, we analyzed our observations by comparing them to the ephemeris positions. This analysis has shown that our observations present a high level of accuracy hardly higher than 50 mas, as it is the average value of the standard deviations of residuals. However, mean residuals are lower, with less than 30 mas in both coordinates, showing the very high accuracy.

For the planet Neptune, we have presented that the ephemeris DE431 appears to be the most homogeneous and accurate as it is the only one presenting mean residuals lower than 30 mas in both coordinates, just followed by INPOP06, nearly as accurate than DE431 in both coordinates, within less than 10 mas. Also DE421, that we have shown to be equivalent to EPM2011m, is in very good agreement with DE431, within less than 20 mas. The other planetary ephemerides as DE405, that we have shown to be equivalent to DE406, INPOP08 and INPOP10 present slightly higher residuals but remain in rather good agreement with DE431, within about 50 mas. Finally, DE200 and VSOP82, the oldest ephemerides, present the highest residuals, up to 900 mas, showing a significant drift of their positions for the recent period of our observations.

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